Wheeler–DeWitt Quantization of Gravity Models of Unified Dark Energy and Dark Matter

Eduardo Guendelman, Emil Nissimov and Svetlana Pacheva

Abstract First, we describe the construction of a new type of gravity-matter models based on the formalism of non-Riemannian space-time volume forms - alternative generally covariant integration measure densities (volume elements) defined in terms of auxiliary antisymmetric tensor gauge fields. Here gravity couples in a non-conventional way to two distinct scalar fields providing a unified Lagrangian action principle description of: (i) the evolution of both "early" and"late" Universe - by the "inflaton" scalar field; (ii) dark energy and dark matter as a unified manifestation of a single material entity - the "darkon" scalar field. A physically very interesting phenomenon occurs when including in addition interactions with the electro-weak model bosonic sector - we obtain a gravity-assisted dynamical generation of electroweak spontaneous gauge symmetry breaking in the post-inflationary "late" Universe, while the Higgs-like scalar remains massless in the "early" Universe. Next, we proceed to the Wheeler–DeWitt minisuperspace quantization of the above models. The "darkon" field plays here the role of cosmological"time". In particular, we show the absence of cosmological space-time singularities.

Keywords Dark energy · Dark matter · Non-Riemannian volume-forms
Electroweak symmetry breaking · Wheeler-DeWitt quantization

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1 Introduction

Among the most important paradigms at the interface of particle physics and cosmology [1–7] one should mention:

- (i) The nature of dark energy and dark matter – both "dark" species occupying around 70 and 25% of the matter content of the "late" (today's) Universe, respectively, continue to be the two most unexplained "mysteries" in cosmology and astrophysics (for a background, see [8–17]).
- (ii) The interplay between the cosmological dynamics and the evolution of the symmetry breaking patterns along the history of the Universe – specifically, for the present epoch's phase of slowly accelerating Universe (dark energy domination) see [8–14], and for a recent general account see [18, 19].

There exist a multitude of proposals for an adequate description of dark energy's and dark matter's dynamics within the framework of standard general relativity or its modern extensions, among them: "Chaplygin gas" models [20–22], "purely kinetic k-essence" models [23, 24], "mimetic" dark matter models [25–28].

Addressing issue (i) above, in Sect. 2 we will briefly review our own approach [29, 30] (for some earlier works, see also [31, 32]) to one of the principal challenge in modern cosmology to understand theoretically from first principles the nature of both "dark" species as a manifestation of the dynamics of a single entity of matter. In the simplest setting we achieve unified description of dark energy and dark matter based on a class of generalized non-canonical models of gravity interacting with a single scalar *"darkon"* field employing the method of non-Riemannian volume-forms on the pertinent spacetime manifold, i.e., non-Riemannian volume elements. Originally [33–35] this approach was proposed as introducing alternative generally covariant integration measure densities in terms of auxiliary "measure" scalar fields. Later [36–38] it was reformulated in a more consistent geometrical setting, namely, the non-Riemannian volume-forms are constructed in terms of auxiliary higher-rank antisymmetric tensor gauge fields, which were shown to be essentially pure-gauge degrees of freedom, i.e., *no* additional propagating field-theoretic (gravitational) degrees of freedom are introduced.

Next, addressing issue (ii) we extend [39, 40] the above non-canonical gravity-matter model by adding coupling to a second scalar *"inflaton"* field describing the universe's evolution in a unified way ("quintessence"), as well as coupling to the fields of the electroweak bosonic sector. In this way we obtain a *gravity-assisted* generation of electro-weak spontaneous gauge symmetry breaking in the post-inflationary "late" Universe, while the Higgs-like scalar remains massless in the "early" Universe [40, 41].

In Sect. 3 we perform Wheeler–DeWitt [42, 43] minisuperspace quantization of the above models. The "darkon" field plays the role of cosmological "time" in the pertinent Wheeler–DeWitt equation in the "early" universe. We show explicitly the absence of cosmological singularities in the wave function of the universe.

2 Quintessence, Unified Dark Energy and Dark Matter, and Gravity-Assisted Higgs Mechanism

2.1 Hidden Noether Symmetry and Unification of Dark Energy and Dark Matter

First, let us consider the following simple particular case of a non-conventional gravity-scalar-field action – a member of the general class of the non-Riemannian-volume-element-based gravity-matter theories [38, 39] (for simplicity we use units with the Newton constant $G_N = 1/16\pi$):

$$S = \int d^4x \sqrt{-g}\, R + \int d^4x \big(\sqrt{-g} + \Phi(C)\big) L(u, Y) \; . \tag{1}$$

Here R denotes the standard Riemannian scalar curvature for the pertinent Riemannian metric $g_{\mu\nu}$. In the second term in (1) – the scalar field Lagrangian is coupled *symmetrically* to two mutually independent spacetime volume-elements – the standard Riemannian $\sqrt{-g}$ and to an alternative non-Riemannian one:

$$\Phi(C) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu C_{\nu\kappa\lambda} \; . \tag{2}$$

$L(u, Y)$ is general-coordinate invariant Lagrangian of a single scalar field $u(x)$, the simplest example being:

$$L(u, Y) = Y - V(u) \quad , \quad Y \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu u \partial_\nu u \; , \tag{3}$$

Crucial new property – we obtain *dynamical constraint* on $L(u, Y)$ as a result of the equations of motion w.r.t. $C_{\mu\nu\lambda}$:

$$\partial_\mu L(u, Y) = 0 \;\longrightarrow\; L(u, Y) = -2M_0 = \text{const} \; , \tag{4}$$

i.e., $Y = V(u) - 2M_0$. M_0 will play the role of dynamically generated cosmological constant.

A second crucial property – *hidden strongly nonlinear Noether symmetry* of scalar field action in (1) – is due to the presence of the non-Riemannian volume element $\Phi(C)$. The scalar field action is invariant (up to a total derivative) under the following nonlinear symmetry transformations:

$$\delta_\epsilon u = \epsilon \sqrt{Y} \quad , \quad \delta_\epsilon g_{\mu\nu} = 0 \quad , \quad \delta_\epsilon \mathcal{C}^\mu = -\epsilon \frac{1}{2\sqrt{Y}} g^{\mu\nu} \partial_\nu u \big(\Phi(C) + \sqrt{-g}\big) \; , \tag{5}$$

where $\mathcal{C}^\mu \equiv \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} C_{\nu\kappa\lambda}$.

Then, standard Noether procedure yields a conserved current:

$$\nabla_\mu J^\mu = 0 \quad , \quad J^\mu \equiv -\Big(1 + \frac{\Phi(C)}{\sqrt{-g}}\Big)\sqrt{2Y}\, g^{\mu\nu}\partial_\nu u \tag{6}$$

The energy-momentum tensor $T_{\mu\nu}$ and J^μ (6) can be cast into a relativistic hydro-dynamical form (taking into account (4)):

$$T_{\mu\nu} = -2M_0 g_{\mu\nu} + \rho_0 u_\mu u_\nu \quad , \quad J^\mu = \rho_0 u^\mu \ , \tag{7}$$

where the pressure $p = -2M_0 = \text{const}$ and:

$$\rho_0 \equiv \Big(1 + \frac{\Phi(C)}{\sqrt{-g}}\Big) 2Y \quad , \quad u_\mu \equiv -\frac{\partial_\mu u}{\sqrt{2Y}} \quad , \quad u^\mu u_\mu = -1 \ . \tag{8}$$

The total energy density is $\rho = \rho_0 - p = 2M_0 + \Big(1 + \frac{\Phi(C)}{\sqrt{-g}}\Big) 2Y$.

Because of the constant pressure ($p = -2M_0$) $\nabla^\nu T_{\mu\nu} = 0$ implies *both* hidden Noether symmetry current $J^\mu = \rho_0 u^\mu$ conservation, as well as *geodesic fluid motion*:

$$\nabla_\mu \big(\rho_0 u^\mu\big) = 0 \quad , \quad u_\nu \nabla^\nu u_\mu = 0 \ . \tag{9}$$

Therefore, $T_{\mu\nu} = -2M_0 g_{\mu\nu} + \rho_0 u_\mu u_\nu$ represents an exact sum of two contributions of the two dark species:

$$p = p_{\rm DE} + p_{\rm DM} \quad , \quad \rho = \rho_{\rm DE} + \rho_{\rm DM} \tag{10}$$

$$p_{\rm DE} = -2M_0 \ , \ \rho_{\rm DE} = 2M_0 \ ; \ p_{\rm DM} = 0 \ , \ \rho_{\rm DM} = \rho_0 \ , \tag{11}$$

i.e., the dark matter component is a dust fluid flowing along geodesics. This is explicit unification of dark energy and dark matter originating from the dynamics of a single scalar field - the “darkon” u.

2.2 Quintessential Inflation and Unified Dark Energy and Dark Matter

We will now extend our previous gravity-“darkon” model to gravity coupled to both “inflaton” $\varphi(x)$ and “darkon” $u(x)$ scalar fields within the non-Riemannian volume-form formalism, as well as we will also add coupling to the bosonic sector of the electro-weak model:

$$S = \int d^4x\, \Phi(A)\Big[g^{\mu\nu}R_{\mu\nu}(\Gamma) + L_1(\varphi, X) + L_2(\sigma, \nabla\sigma; \varphi)\Big] + \int d^4x\, \Phi(B)\Big[U(\varphi) + L_3(\mathcal{A}, \mathcal{B}) + \frac{\Phi(H)}{\sqrt{-g}}\Big] + \int d^4x \big(\sqrt{-g} + \Phi(C)\big) L(u, Y) \ . \tag{12}$$

Here the following notations are used:

- $\Phi(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda}$ and $\Phi(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda}$ – two new independent non-Riemannian volume-forms (non-Riemannian volume elements) apart from $\Phi(C)$;
- $\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}$ is the dual field-strength of an additional auxiliary tensor gauge field $H_{\nu\kappa\lambda}$ crucial for the consistency of (12).
- Important – we use Palatini formalism: $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$; $g_{\mu\nu}$, $\Gamma^\lambda_{\mu\nu}$ – metric and affine connection are apriori independent.
- $\sigma \equiv (\sigma_a)$ is a complex $SU(2)\times U(1)$ iso-doublet Higgs-like scalar field with a Lagrangian:

$$L_2(\sigma, \nabla\sigma; \varphi) = -g^{\mu\nu}\big(\nabla_\mu\sigma_a\big)^*\nabla_\nu\sigma_a - V_0(\sigma)e^{\alpha\varphi} \; . \tag{13}$$

The gauge-covariant derivative acting on σ reads:

$$\nabla_\mu\sigma = \Big(\partial_\mu - \frac{i}{2}\tau_A\mathcal{A}^A_\mu - \frac{i}{2}\mathcal{B}_\mu\Big)\sigma \; , \tag{14}$$

with $\frac{1}{2}\tau_A$ (τ_A – Pauli matrices, $A = 1, 2, 3$) indicating the $SU(2)$ generators.

- The “bare” σ-field potential is of the same form as the standard Higgs potential:

$$V_0(\sigma) = \frac{\lambda}{4}\left((\sigma_a)^*\sigma_a - \mu^2\right)^2 \; . \tag{15}$$

- The $SU(2)\times U(1)$ gauge field action $L(\mathcal{A}, \mathcal{B})$ is of the standard Yang–Mills form (all $SU(2)$ indices $A, B, C = (1, 2, 3)$):

$$L_3(\mathcal{A}, \mathcal{B}) = -\frac{1}{4g^2}F^2(\mathcal{A}) - \frac{1}{4g'^2}F^2(\mathcal{B}) \; , \tag{16}$$

$$F^2(\mathcal{A}) \equiv F^A_{\mu\nu}(\mathcal{A})F^A_{\kappa\lambda}(\mathcal{A})g^{\mu\kappa}g^{\nu\lambda} \;\; , \;\; F^2(\mathcal{B}) \equiv F_{\mu\nu}(\mathcal{B})F_{\kappa\lambda}(\mathcal{B})g^{\mu\kappa}g^{\nu\lambda} \; ,$$
$$F^A_{\mu\nu}(\mathcal{A}) = \partial_\mu\mathcal{A}^A_\nu - \partial_\nu\mathcal{A}^A_\mu + \epsilon^{ABC}\mathcal{A}^B_\mu\mathcal{A}^C_\nu \;\; , \;\; F_{\mu\nu}(\mathcal{B}) = \partial_\mu\mathcal{B}_\nu - \partial_\nu\mathcal{B}_\mu \; .$$

$\mathcal{A}^A_\mu$ ($A = 1, 2, 3$) and $\mathcal{B}_\mu$ denote the corresponding $SU(2)$ and $U(1)$ electroweak gauge fields.

- The “inflaton” φ Lagrangian terms are given by:

$$L_1(\varphi, X) = X - V_1(\varphi) \;\; , \;\; X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \; , \tag{17}$$

$$V_1(\varphi) = f_1\exp\{\alpha\varphi\} \;\; , \;\; U(\varphi) \equiv f_2\exp\{2\alpha\varphi\} \; , \tag{18}$$

where α, f_1, f_2 are dimensionful positive parameters.

- The form of the action (12) is fixed by the requirement of invariance under global Weyl-scale transformations:

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \; , \; \Gamma^{\mu}_{\nu\lambda} \to \Gamma^{\mu}_{\nu\lambda} \; , \; \varphi \to \varphi - \frac{1}{\alpha}\ln\lambda \; ,$$
$$A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \; , \; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \; , \; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} \; , \tag{19}$$

and the electro-weak sector $(\sigma, \mathcal{A}, \mathcal{B})$ is inert w.r.t. (19).

Equations of motion w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$ yield a solution for the latter as a Levi-Civita connection:

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda}\right) \; , \tag{20}$$

w.r.t. to the *Weyl-rescaled metric* $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu} \quad , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \; . \tag{21}$$

Transition from original metric $g_{\mu\nu}$ to $\bar{g}_{\mu\nu}$: *"Einstein-frame"*, where the gravity equations of motion are written in the standard form of Einstein's equations: $R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\rm eff}_{\mu\nu}$ with an appropriate *effective* energy-momentum tensor given in terms of an Einstein-frame matter Lagrangian $L_{\rm eff}$ (see (25) below).

Solutions of the eqs. of motion of the action (12) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yield:

$$\frac{\Phi(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} \quad , \quad R + L_1(\varphi, X) + L_2(\sigma, \nabla\sigma; \varphi) = M_1 = \text{const} \; ,$$
$$U(\varphi) + L_3(\mathcal{A}, \mathcal{B}) + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const} \; . \tag{22}$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants, similar to M_0 (4).

Within the canonical Hamilton formalism we have shown [37, 38, 44] that M_0, $M_{1,2}$, χ_2 are the only remnant of the auxiliary gauge fields $C_{\mu\nu\lambda}$, $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$, $H_{\mu\nu\lambda}$ entering (12) – they have the meaning of conserved Dirac-constrained canonical momenta conjugated to some of the components of the latter.

We derive from (12) the physical *Einstein-frame* theory w.r.t. Weyl-rescaled Einstein-frame metric $\bar{g}_{\mu\nu}$ (21) and perform an additional "darkon" field redefinition $u \to \widetilde{u}$:

$$\frac{\partial \widetilde{u}}{\partial u} = \left(V_1(u) - 2M_0\right)^{-\frac{1}{2}} \quad ; \quad Y \to \widetilde{Y} = -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu \widetilde{u}\partial_\nu \widetilde{u} \; . \tag{23}$$

The Einstein-frame action reads:

$$S = \int d^4x \sqrt{-\bar{g}}\Big[R(\bar{g}) + L_{\rm eff}\big(\varphi, \bar{X}, \widetilde{Y}; \sigma, \bar{X}_\sigma, \mathcal{A}, \mathcal{B}\big)\Big] \, , \tag{24}$$

where (now the kinetic terms are given in terms of the Einstein-frame metric (21), e.g. $\bar{X} = -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$, etc.):

$$L_{\rm eff}\big(\varphi, \bar{X}, \widetilde{Y}; \sigma, \bar{X}_\sigma, \mathcal{A}, \mathcal{B}\big) = \bar{X} - \widetilde{Y}\Big(V_1(\varphi) + V_0(\sigma)e^{\alpha\varphi} + M_1\Big)$$
$$+\widetilde{Y}^2\Big[\chi_2(U(\varphi) + M_2) - 2M_0\Big] + L[\sigma, \bar{X}_\sigma, \mathcal{A}, \mathcal{B}] \,, \tag{25}$$

with $L[\sigma, \bar{X}_\sigma, \mathcal{A}, \mathcal{B}] \equiv -\bar{g}^{\mu\nu}\big(\nabla_\mu\sigma_a\big)^*\nabla_\nu\sigma_a - \frac{\chi_2}{4g^2}\bar{F}^2(\mathcal{A}) - \frac{\chi_2}{4g'^2}\bar{F}^2(\mathcal{B})$.

For static (spacetime independent) scalar field configurations we obtain from (25) the following Einstein-frame effective scalar "inflaton+Higgs" effective potential:

$$U_{\rm eff}\big(\varphi, \sigma\big) = \frac{\Big(V_1(\varphi) + V_0(\sigma)e^{\alpha\varphi} + M_1\Big)^2}{4\big[\chi_2(U(\varphi) + M_2) - 2M_0\big]}$$
$$= \frac{\Big[\Big(f_1 + \frac{\lambda}{4}\big((\sigma_a)^*\sigma_a - \mu^2\big)^2\Big)e^{\alpha\varphi} + M_1\Big]^2}{4\big[\chi_2(f_2 e^{2\alpha\varphi} + M_2) - 2M_0\big]} \,. \tag{26}$$

$U_{\rm eff}\big(\varphi, \sigma\big)$ has few remarkable properties. First, $U_{\rm eff}\big(\varphi, \sigma\big)$ possesses two infinitely large flat regions as function of φ (when σ is fixed):
(a) $(-)$ flat region for large negative values of the "inflaton" φ;
(b) $(+)$ flat region and large positive values of φ,
respectively, as depicted in Fig. 1.

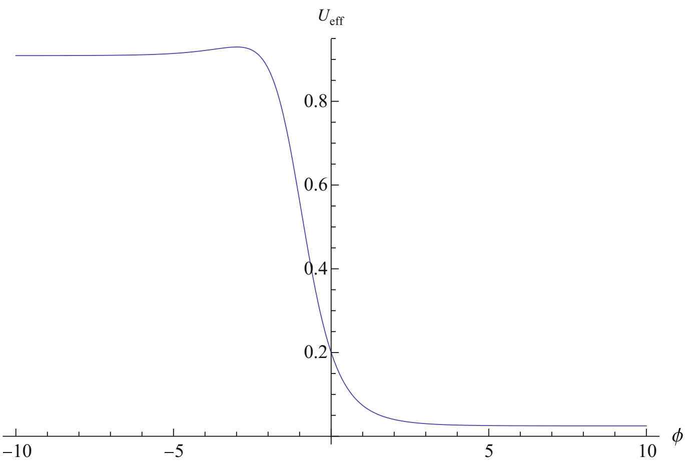

Fig. 1 Qualitative shape of the effective scalar potential $U_{\rm eff}$ (26) as function of φ at σ = fixed for $M_1 > 0$

- In the (+) flat region (large positive "inflaton" values) (26) reduces to:

$$U_{\text{eff}}\big(\varphi,\sigma\big) \simeq U_{(+)}(\sigma) = \frac{\Big(\frac{\lambda}{4}\left((\sigma_a)^*\sigma_a - \mu^2\right)^2 + f_1\Big)^2}{4\chi_2 f_2} \ . \tag{27}$$

- Equation (27) yields as a lowest lying vacuum the Higgs one:

$$|\sigma| = \mu \ , \tag{28}$$

i.e., we obtain the standard spontaneous breakdown of $SU(2) \times U(1)$ gauge symmetry.
- At the Higgs vacuum (28) we get from (27) a dynamically generated cosmological constant $\Lambda_{(+)}$:

$$U_{(+)}(\mu) \equiv 2\Lambda_{(+)} = \frac{f_1^2}{4\chi_2 f_2} \ . \tag{29}$$

- If we identify the integration constants in (26) with the fundamental constants of Nature – M_{Pl} (Planck mass) and M_{EW} (electro-weak mass scale) as $f_1 \sim M_{EW}^4$, $f_2 \sim M_{Pl}^4$, we are then naturally led to a very small vacuum energy density:

$$U_{(+)}(\mu) \sim M_{EW}^8/M_{Pl}^4 \sim 10^{-122} M_{Pl}^4 \ , \tag{30}$$

which is the right order of magnitude for the present epoch's vacuum energy density according to [45]. Therefore, we can identify the (+) flat region (large positive "inflaton" values) of U_{eff} (26) as describing the present "late" universe.

- In the (−) flat region (large negative "inflaton" values) (26) reduces to:

$$U_{\text{eff}}\big(\varphi,\sigma\big) \simeq U_{(-)} \equiv \frac{M_1^2}{4(\chi_2 M_2 - 2M_0)} \ . \tag{31}$$

If we take $M_1 \sim M_2 \sim 10^{-8} M_{Pl}^4$ and $M_0 \sim M_{EW}^4$, then the vacuum energy density $U_{(-)}$ (31) becomes $U_{(-)} \sim 10^{-8} M_{Pl}^4$, which conforms to the Planck Collaboration data [46, 47] for the energy scale of inflation (of order $10^{-2} M_{Pl}$). This allows to identify the (−) flat region (large negative "inflaton" values) of the "inflaton+Higgs" effective potential (26) as describing the"early" universe, in particular, the inflationary epoch.
- In the (−) flat region the effective potential (31) is σ-field idependent. Thus, the Higgs-like iso-doublet scalar field σ_a remains *massless* in the "early" (inflationary) Universe and accordingly there is *no electro-weak spontaneous symmetry breaking* there.

3 Wheeler–De Witt Minisuperspace Quantization

For simplicity here we will consider the unified dark energy/dark matter "quintessential" model (12) without the coupling to the bosonic electro-weak sector. The corresponding Einstein-frame action reads:

$$S = \int d^4x \sqrt{-\bar{g}} \Big[R(\bar{g}) + L_{\rm eff}\big(\varphi, \bar{X}, \widetilde{Y}\big) \Big] \, , \tag{32}$$

where (recall $\bar{X} = -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ and $\widetilde{Y} = -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\widetilde{u}\partial_\nu\widetilde{u}$):

$$L_{\rm eff}\big(\varphi, \bar{X}, \widetilde{Y}\big) = \bar{X} - \widetilde{Y}\Big(V(\varphi) - M_1\Big) + \widetilde{Y}^2 \Big[\chi_2 (U(\varphi) + M_2) - 2M_0 \Big] \, , \tag{33}$$

To study cosmological implications of (32) we perform a Friedmann–Lemaitre–Robertson–Walker (*FLRW*) reduction to the class of FLRW metrics:

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -N^2(t) dt^2 + a^2(t) d\mathbf{x}.d\mathbf{x} \tag{34}$$

and take the "inflaton" and"darkon" to be time-dependent only, i.e.:

$$\bar{X} = \frac{1}{2}\,\dot{\varphi}^2 \quad , \quad \widetilde{Y} = \frac{1}{2} w^2 \quad , \quad w \equiv \frac{d\widetilde{u}}{dt} \, . \tag{35}$$

The FLRW reduced action corresponding to (32) reads:

$$S_{\rm FLRW} = \int dt \left\{ -\frac{1}{N} 6a\ \dot{a}^2 + N a^3 \Big[\frac{\dot{\varphi}^2}{2N^2} - (f_1 e^{\alpha\varphi} + M_1) \frac{w^2}{2N^2} \right. \tag{36}$$

$$\left. + \Big(\chi_2 (f_2 e^{2\alpha\varphi} + M_2) - 2M_0 \Big) \frac{w^4}{4N^4} \Big] \right\} \tag{37}$$

Calculating the canonically conjugated momenta $p_a, p_\varphi, p_{\widetilde{u}}$, we arrive at the canonical FLRW Hamiltonian:

$$\mathcal{H} = N \mathcal{H}_{WDW} = N \left\{ -\frac{p_a^2}{24a} + \frac{p_\varphi^2}{a^3} + p_{\widetilde{u}} w \right. \tag{38}$$

$$\left. + a^3 \Big[(f_1 e^{\alpha\varphi} + M_1) \frac{w^2}{2} - \Big(\chi_2 (f_2 e^{2\alpha\varphi} + M_2) - 2M_0 \Big) \frac{w^4}{4} \Big] \right\} \tag{39}$$

$\mathcal{H}$ turns out to be pure first-class constraint $\mathcal{H}_{WDW}$ a'la Dirac with the lapse N as Lagrange multiplier.

In (39) the "darkon" velocity w is determined as function of the canonical variables $(a, \varphi, p_{\widetilde{u}})$ being the real root (for all values of $(a, \varphi, p_{\widetilde{u}})$) of the cubic algebraic equation:

$$w^3 - 3A(\varphi)\, w - 2\frac{B(\varphi, p_{\widetilde{u}})}{a^3} = 0 \tag{40}$$

where the coefficients are given by:

$$A(\varphi) \equiv \frac{1}{3}\,\frac{(f_1 e^{\alpha\varphi} + M_1)}{\chi_2(f_2 e^{2\alpha\varphi} + M_2) - 2M_0}\,,$$
$$B(\varphi, p_{\widetilde{u}}) \equiv \frac{p_{\widetilde{u}}}{2}\,\frac{1}{\chi_2(f_2 e^{2\alpha\varphi} + M_2) - 2M_0}\,. \tag{41}$$

The solution of (40) for $w = w(a, \varphi, p_{\widetilde{u}})$ reads:

$$w = \mathrm{sign}(B(\varphi, p_{\widetilde{u}}))|A(\varphi)|^{1/2}|\xi|^{-1/6}\Big[\big(1 + \sqrt{1-\xi}\big)^{1/3} + \big(1 - \sqrt{1-\xi}\big)^{1/3}\Big] \tag{42}$$

where $\xi \equiv \xi(a, \varphi, p_{\widetilde{u}}) = \frac{A^3(\varphi)}{9B^2(\varphi, p_{\widetilde{u}})}\, a^6$.

Quantization of the Dirac-constrained canonical Hamilton (39) yields the Wheeler–DeWitt (WDW) equation for the wave function of the universe $\Psi = \Psi(a, \varphi; p_{\widetilde{u}})$:

$$\widehat{\mathcal{H}}_{WDW}\Psi(a, \varphi; p_{\widetilde{u}}) = 0\,, \tag{43}$$

where $\widehat{\mathcal{H}}_{WDW}$ is the quantum version of $\mathcal{H}_{WDW}$ in (39). We resolve the ordering ambiguity there by changing variables:

$$a \to \widetilde{a} = \frac{4}{\sqrt{3}} a^{3/2}\,, \tag{44}$$

and taking the special operator ordering:

$$\frac{p_a^2}{24a} \to \frac{1}{2}\frac{1}{\sqrt{12a}}\widehat{p}_a\frac{1}{\sqrt{12a}}\widehat{p}_a = -\frac{1}{2}\frac{\partial^2}{\partial\widetilde{a}^2}\,. \tag{45}$$

The WDW operator $\widehat{\mathcal{H}}_{WDW}$ becomes:

$$\widehat{\mathcal{H}}_{WDW} = \frac{1}{2}\frac{\partial^2}{\partial\widetilde{a}^2} + \frac{8}{3\widetilde{a}^2}\widehat{p}_\varphi^2 + \frac{3}{4}p_{\widetilde{u}}w + \frac{3}{64}w^2\widetilde{a}^2(f_1 e^{\alpha\varphi} + M_1)\,, \tag{46}$$

where $\widehat{p}_\varphi = -i\partial/\partial\varphi$ and $w = w(\widetilde{a}, \varphi, p_{\widetilde{u}})$ is the solution (42) of the cubic equation (40).

The final form of WDW equation reads:

$$\Big[\frac{1}{2}\Big(\frac{\partial}{\partial\widetilde{a}}\Big)^2 + \frac{8}{3\widetilde{a}^2}\widehat{p}_\varphi^2 + U(\widetilde{a}, \varphi, p_{\widetilde{u}})\Big]\Psi(\widetilde{a}, \varphi; p_{\widetilde{u}}) = 0\,, \tag{47}$$

$$U(\widetilde{a}, \varphi, p_{\widetilde{u}}) \equiv \frac{\widetilde{a}^2(f_1 e^{\alpha\varphi} + M_1)^2}{64(\chi_2 f_2 e^{2\alpha\varphi} + \chi_2 M_2 - 2M_0)}\mathcal{F}\big(\xi(\widetilde{a}, \varphi, p_{\widetilde{u}})\big)) \tag{48}$$

with the following notations:

$$\xi(\widetilde{a}, \varphi, p_{\widetilde{u}}) \equiv \frac{\widetilde{a}^4 (f_1 e^{\alpha\varphi} + M_1)^3}{192 p_{\widetilde{u}}^2 (\chi_2 f_2 e^{2\alpha\varphi} + \chi_2 M_2 - 2M_0)} , \tag{49}$$

$$\mathcal{F}(\xi) \equiv \xi^{-1/3}\Big[\big(1+\sqrt{1-\xi}\big)^{1/3} + \big(1-\sqrt{1-\xi}\big)^{1/3}\Big] \times \Big[2\xi^{-1/3} + \big(1+\sqrt{1-\xi}\big)^{1/3} + \big(1-\sqrt{1-\xi}\big)^{1/3}\Big] . \tag{50}$$

Analytic solutions of (47) can be found when the "inflaton" φ is either on the $(-)$ flat region (φ large negative – "early" universe) or on $(+)$ flat region (φ large positive – "late"/nowadays universe), cf. Fig. 1 above.

In the $(+)$ flat region of the "inflaton" φ ("late" universe) the WDW equation (47) reduces to the quantum mechanical Schrödinger equation:

$$\Big[\frac{1}{2}\frac{\partial^2}{\partial\widetilde{a}^2} + \mathcal{W}_{(+)}(\widetilde{a}, p_\varphi)\Big]\Psi(\widetilde{a}, p_\varphi) = 0 , \tag{51}$$

$$\mathcal{W}_{(+)}(\widetilde{a}, p_\varphi) \equiv \frac{3 f_1^2}{64\chi_2 f_2}\widetilde{a}^2 + \frac{8 p_\varphi^2}{3}\widetilde{a}^{-2} \quad , \quad p_\varphi - \text{small} . \tag{52}$$

The solution of (51) reads (here $c_{1,2}$ are constants):

$$\Psi(\widetilde{a}, p_\varphi) = \sqrt{\widetilde{a}}\Big[c_1 J_{\frac{1}{4}\sqrt{1-\gamma}}\big(\frac{1}{2}\beta\widetilde{a}^2\big) + c_2 J_{-\frac{1}{4}\sqrt{1-\gamma}}\big(\frac{1}{2}\beta\widetilde{a}^2\big)\Big] , \tag{53}$$

$$\beta \equiv \sqrt{\frac{3 f_1^2}{32\chi_2 f_2}} \quad , \quad \gamma \equiv \frac{64}{3} p_\varphi^2 \quad (\gamma - \text{small}) , \tag{54}$$

$$\Psi(\widetilde{a}, p_\varphi) \simeq \text{const}\ \widetilde{a}^{\frac{1}{2}(1-\sqrt{1-\gamma})} \quad \text{for} \quad \widetilde{a} \to 0 , \tag{55}$$

i.e., the wave function (53) vanishes at $\widetilde{a} = 0$.

Similarly, in the $(-)$ flat region of the "inflaton" φ ("early" universe) the WDW equation (47) reduces to the quantum mechanical Schrödinger equation:

$$\Big[\frac{1}{2}\frac{\partial^2}{\partial\widetilde{a}^2} + \mathcal{W}_{(-)}(\widetilde{a}, p_\varphi, p_{\widetilde{u}})\Big]\Psi(\widetilde{a}, p_\varphi, p_{\widetilde{u}}) = 0 , \tag{56}$$

$$\mathcal{W}_{(-)}(\widetilde{a}, p_\varphi, p_{\widetilde{u}}) = \frac{3M_1^2}{64(\chi_2 M_2 - 2M_0)}\widetilde{a}^2 + \frac{8 p_\varphi^2}{3}\widetilde{a}^{-2} + p_{\widetilde{u}}\sqrt{\frac{M_1}{\chi_2 M_2 - 2M_0}} + O\Big(\frac{p_{\widetilde{u}}^2}{\widetilde{a}^2}\Big) . \tag{57}$$

In (56) and (57) the canonical "darkon" momentum (times a constant) plays the role of energy eigenvalue $E \equiv p_{\widetilde{u}}\sqrt{\frac{M_1}{\chi_2 M_2 - 2M_0)}}$, meaning that the "darkon" field $\widetilde{u}$ plays the role of cosmological"time" in the "early" universe.

We can solve explicitly WDW equation (56) for small "darkon" momenta $p_{\widetilde{u}}$ ignoring the last term in (57):

$$\Psi(\widetilde{a}, p_\varphi, p_{\widetilde{u}}) = \text{const}\ \widetilde{a}^{\frac{1}{2}(1+\sqrt{1-\gamma})}\, e^{\frac{i}{2}\beta\widetilde{a}^2} \times U\Big(\frac{1}{4}(2+\sqrt{1-\gamma}) - i\frac{E}{2\beta}, \frac{1}{2}(2+\sqrt{1-\gamma}); -i\beta\widetilde{a}^2\Big)\ , \qquad (58)$$

$$\beta \equiv \sqrt{\frac{3M_1^2}{32(\chi_2 M_2 - 2M_0}}\quad ,\quad \gamma \equiv \frac{64}{3}p_\varphi^2\quad ,\quad E \equiv p_{\widetilde{u}}\sqrt{\frac{M_1}{\chi_2 M_2 - 2M_0}}\ , \qquad (59)$$

where $U(\cdot, \cdot; z)$ denotes the confluent hypergeometric function of the second kind. Again as in (55) the wave function (58) vanishes at $\widetilde{a} = 0$:

$$\Psi(\widetilde{a}, p_\varphi, p_{\widetilde{u}}) \simeq \text{const}\ \widetilde{a}^{\frac{1}{2}(1-\sqrt{1-\gamma})}\quad \text{for}\quad \widetilde{a} \to 0\ , \qquad (60)$$

In the inflationary "slow-roll" regime in the"early" Universe the "inflaton" canonical momentum p_φ is very small. Thus, ignoring also the second term in $\mathcal{W}_{(-)}$ (57) and Fourier-transforming (58) w.r.t. canonical "darkon" momentum $p_{\widetilde{u}}$ with E as in (59):

$$\Psi(\widetilde{a}, \tau) = \int_{-\infty}^{\infty}\frac{dE}{2\pi}\Psi(\widetilde{a}, p_\varphi\!=\!0, p_{\widetilde{u}})\, e^{-iE\tau}\quad ,\quad E \equiv p_{\widetilde{u}}\sqrt{\frac{M_1}{\chi_2 M_2 - 2M_0}}\ , \qquad (61)$$

i.e., $\tau \sim \widetilde{u}$ being the "cosmological" time, the WDW equation (56) and (57) acquires the form of a time-dependent Schrödinger equation for the inverted harmonic oscillator:

$$i\frac{\partial}{\partial\tau}\Psi(\widetilde{a}, \tau) = \Big[-\frac{1}{2}\frac{\partial^2}{\partial\widetilde{a}^2} - \omega^2\widetilde{a}^2\Big]\Psi(\widetilde{a}, \tau) \qquad (62)$$

with a negative "frequency" squared:

$$-\omega^2 \equiv -\frac{3M_1^2}{64(\chi_2 M_2 - 2M_0)} \equiv -\frac{3}{16}U_{(-)}\ , \qquad (63)$$

where $U_{(-)}$ (31) is the vacuum energy density of the inflationary epoch.

The solution of equation (62) in the form of a normalized (on the semiaxis $\widetilde{a} \in (0, \infty)$) wave packet has already been found in [48]:

$$\Psi(\widetilde{a}, \tau) = \Big(\frac{2\omega}{\pi}\sin(2b)\Big)^{1/4}\big(\cos(b - i\omega\tau)\big)^{-1/2} \times \exp\Big\{-\frac{1}{2}\widetilde{a}^2\omega\tan(b - i\omega\tau)\Big\}\ , \qquad (64)$$

where the parameter b describes the width of the wave packet. Calculating the average value of the FLRW scale factor $a = \frac{\sqrt{3}}{4}\widetilde{a}^{2/3}$ (cf. (44)) we obtain:

$$\langle\widetilde{a}\rangle \equiv \int_0^\infty d\widetilde{a}\,\widetilde{a}|\Psi(\widetilde{a},\tau)|^2 = \left[\frac{\cos(2b) + \cosh(2\omega\tau)}{\pi\omega\sin(2b)}\right]^{1/2} . \tag{65}$$

Thus, the quantum average of the FLRW scale factor does not exhibit any singularity ($\langle\widetilde{a}\rangle \to 0$) at any "time" τ.

4 Conclusions

Employing non-Riemannian spacetime volume-forms (non-Riemannian volume elements) in generalized gravity-matter theories allows for several interesting developments:

- Simple unified description of dark energy and dark matter as manifestation of the dynamics of a single non-canonical scalar field ("darkon").
- Construction of a new class of models of gravity interacting with a scalar "inflaton" φ, as well as with other phenomenologically relevant matter including Higgs-like scalar σ, which produce an effective full scalar potential of φ, σ with few remarkable properties.
- The "inflaton" effective potential (at fixed σ) possesses two infinitely large flat regions with vastly different energy scales for large negative and large positive values of φ. This allows for a unified description of both "early" universe inflation as well as of present "dark energy"-dominated epoch in universe's evolution.
- In the "early" universe the would-be Higgs field σ remains massless and decouples from the "inflaton" φ. The"early" universe evolution is described entirely in terms of the "inflaton" dynamics.
- In the post-inflationary epoch φ and σ exchange roles. The inflaton φ becomes massless and decoupled, whereas σ becomes a genuine Higgs field with a dynamically generated electro-weak-type symmetry breaking effective potential.
- A natural choice for the parameters involved conforms to quintessential cosmological and electro-weak phenomenologies.
- Minisuperspace Wheeler–DeWitt quantization reveals the role of the "darkon" scalar field as cosmological"time" in the "early" Universe. The quantum average of the FLRW scale factor does not exhibit any singularity in its "time" evolution.

Let us also note that applying the non-Riemannian volume-form formalism to minimal $N = 1$ supergravity we arrived at a novel mechanism for the supersymmetric Brout-Englert-Higgs effect, namely, the appearance of a dynamically generated cosmological constant triggering spontaneous supersymmetry breaking and mass generation for the gravitino [36, 44]. Applying the same non-Riemannian volume-form formalism to anti-de Sitter supergravity produces simultaneously a very large physical gravitino mass and a very small *positive* observable cosmological constant [36, 44] in accordance with modern cosmological scenarios for slowly expanding universe of the present epoch [8–14].

As a final comment let us mention some further extensions of the method of non-Riemannian volume elements – gravity models with dynamical spacetime [49] further developed into models of interacting diffusive unified dark energy and dark matter (see [50] and references therein).

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